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On self-similar blast waves headed by the Chapman–Jouguet detonation

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The paper explores the whole class of self-similar solutions for blast waves bounded by Chapman–Jouguet detonations that propagate into a uniform, quiescent, zero counter-pressure atmosphere of a perfect gas with constant specific heats. Since such conditions can be approached quite closely by some actual chemical systems at N.T.P., this raises the interesting possibility of the existence of Chapman–Jouguet detonations of variable velocity. The principal virtue of the results presented here is, however, more of theoretical significance. They represent the limiting case for all the self-similar blast waves headed by gasdynamic discontinuities associated with a deposition of finite amounts of energy, and they exhibit some unique features owing to the singular nature of the Chapman–Jouguet condition.

1. Introduction

In our previous paper on self-similar blast waves (Oppenheim *et al.* 1972), it was shown that the Hugoniot curves on the phase plane of reduced blast wave variables

$$F \equiv \frac{t}{r\mu}u, \quad Z \equiv \left(\frac{t}{r\mu}a\right)^2, \quad \text{where} \quad \mu \equiv \frac{d\ln r_n}{d\ln t_n} \tag{1}$$

(the symbols t, r, u and a representing, respectively, the time and space coordinates, particle velocity and sound speed, while the subscript n refers to the front), approach rapidly the limiting case of zero counter pressure for quite reasonable values of the Hugoniot constant. The latter is expressed most conveniently in terms of the pressure ratio $P_G \equiv p_G/p_0$, where G is the point on the Hugoniot curve corresponding to initial density. Thus, as is illustrated on figure 1, the Hugoniot curve on the phase plane for $P_G = 10$ is already almost coincident with that for $P_G = \infty$. Representative values of P_G for some typical chemical systems are listed in table 1. It appears then that, if in most of these systems a blast wave headed by a Chapman–Jouguet detonation is formed, the boundary condition for such a flow field is given by the intersection of the $P_G = \infty$ line and the locus of the sonic condition, the $D = 0$ parabola.

Solutions corresponding to this boundary condition, CJ_∞, have singularly unique properties. The purpose of this paper is to examine salient features of such solutions for the case of self-similar blast waves propagating into a uniform atmosphere of a perfect gas with constant specific heats at rest.

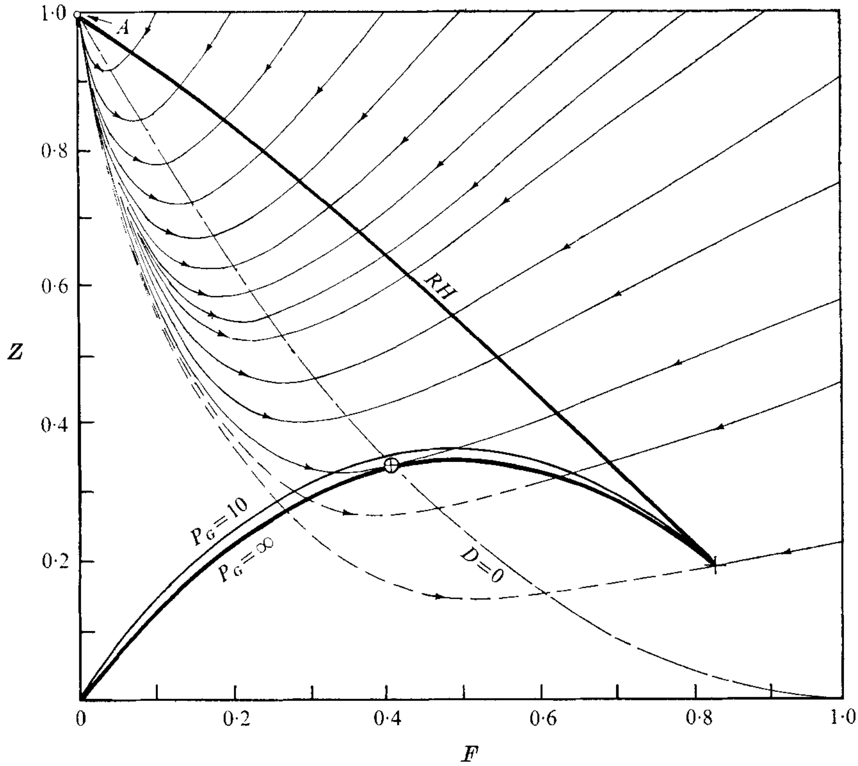

FIGURE 1. Phase plane for spherical waves with constant front velocity ($j = 2$, $\lambda = 0$, $\gamma = 1{\cdot}4$).

Chemical system	P_G
$H_2 + \frac{1}{2}O_2$	9·558
$C_2H_2 + O_2$	23·052
$C_2H_2 + 2\frac{1}{2}O_2$	17·086
$CH_4 + 2O_2$	14·876
$C_2H_6 + 3\frac{1}{2}O_2$	17·193
$C_3H_8 + 5O_2$	18·299
CH_4 + Stoichiometric air	8·809
C_2H_2 + Stoichiometric air	9·777
C_2H_4 + Stoichiometric air	9·395

TABLE 1. Representative values of P_G for a variety of chemical systems at N.T.P.

In this connexion it should be noted that, although the treatment of self-similar blast waves headed by CJ detonations was initiated by Taylor (1950) and, indeed, included in the texts of Sedov (1959), Zel'dovich & Kompaneets (1960) and Zel'dovich & Raizer (1966), the studies described there were confined to constant-velocity waves only. The present paper exposes the existence of solutions associated with Chapman–Jouguet detonations of variable velocity: the case that, undoubtedly on account of its somewhat paradoxical nature, has so far escaped the attention of research workers in this field of study. Nonetheless, as is demonstrated by the values of P_G in table 1, which are in effect equivalent to $P_G = \infty$, the practical significance of such cases is certainly within the realm of physical possibility. What makes the subject of our paper particularly interesting, however, is the fact that it represents a limiting and singular case for all possible self-similar blast waves headed by strong discontinuities. In this respect it complements the treatment presented in our previous paper (Oppenheim *et al.* 1972) on the class of solutions corresponding to strong shocks. Between the two are such cases as that corresponding to a constant rate of energy addition at the front, representing strong blast waves driven by laser irradiation, which has been recently analysed by Champetier, Couairon & Vendenboomgaerde (1968) and by Wilson & Turcotte (1970).

2. Governing equations and boundary conditions

As was demonstrated in our previous paper (Oppenheim *et al.* 1972), propagation of self-similar blast waves into a uniform atmosphere of a perfect gas with constant γ at rest is governed by the differential equation

$$\frac{dZ}{dF} = \frac{Z}{1-F}\frac{P(F,Z)}{Q(F,Z)}, \tag{2}$$

where $Q(F,Z) \equiv (j+1)[F-\lambda/(j+1)\gamma]Z-(\tfrac{1}{2}\lambda+1-F)(1-F)F$ (3)

and $P(F,Z) \equiv [\lambda+2-\{(j+1)(\gamma-1)+2\}F]D(F,Z)+(\gamma-1)Q(F,Z),$ (4)

while $D(F,Z) \equiv Z-(1-F)^2.$ (5)

In the above $\lambda \equiv -2d\ln w_n/d\ln r_n$ is the so-called decay parameter, which is related to the μ of (1) by the relation

$$\lambda = 2\left[\frac{1-\mu}{\mu}-\frac{d\ln\mu}{d\ln r_n}\right], \tag{6}$$

while w_n is the propagation velocity of the wave front. Geometry is taken into account by the integer j, which is equal to 0, 1 and 2 for plane-, line- and point-symmetrical waves, respectively. Positive λ's correspond to decaying waves and negative λ's to accelerating waves. The case of $\lambda = 0$ corresponds to waves having constant front velocity. Since, according to classical concepts, the propagation speed of a Chapman–Jouguet detonation is essentially invariant, all the solutions reported in the literature are concerned with representative cases of such waves only. In order to provide a concise résumé of this class of problems salient properties of blast waves headed by detonation fronts of constant velocity are considered first.

A set of integral curves for this case is shown in figure 1; a similar set, although not to scale, is included in the text of Courant & Friedrichs (1948). The curves were determined by a straightforward numerical integration of (2) using (3), (4) and (5) with $j = 2$, $\lambda = 0$ and $\gamma = 1{\cdot}4$. Each of the curves represents a solution specifying the structure of a blast wave. The arrows denote the direction of increasing radius and change sense at the $D = 0$ line, which thus forms a boundary separating two classes of problems. Those to the right of it are piston-driven, the locus of conditions at the piston face being given by the $F = 1$ line. They can terminate at any point between the Rankine–Hugoniot curve RH and the $D = 0$ line, depending on whether the front of the wave is a shock (end-point on the RH line), a Chapman–Jouguet detonation (end-point on the $D = 0$ line) or a strong detonation (any point between the two). Those to the left are point explosions headed by the Chapman–Jouguet detonation (end-point on the $D = 0$ line). The possibility of a weak detonation acting as a front of a blast wave (the end-point being anywhere to the left of the $D = 0$ line) is ruled out as physically improbable. For this reason also certain portions of integral curves are represented by broken lines to indicate that they correspond to physically unrealistic conditions. The extent of the regime of boundary conditions is terminated by the point of intersection between the $P_G = \infty$ line and the Rankine–Hugoniot curve marked by a cross. It represents the conditions immediately behind a shock whose Mach number is infinite.

Boundary conditions for the class of problems considered here in detail are given by the co-ordinates of the point CJ_∞. This point lies at the intersection of the $P_G = \infty$ line given by the equation

$$Z = \gamma(1-F)F \tag{7}$$

and the locus of Chapman–Jouguet states on the line $D = 0$:

$$Z = (1-F)^2. \tag{8}$$

From the above, one obtains

$$F_n = 1/(\gamma+1), \quad Z_n = [\gamma/(\gamma+1)]^2. \tag{9}$$

An interesting property of the CJ_∞ point is that the slope of the integral curve in the phase plane is at this point independent of λ and j, being a function of γ only, that is, as a consequence of (2)–(5) and (9),

$$\left.\frac{dZ}{dF}\right|_{CJ_\infty} = \gamma\frac{\gamma-1}{\gamma+1}.$$

Once the integral curve $Z = Z(F)$ of (2) has been determined, the position in the flow field of a given state, specified in terms of F and Z, is evaluated by the quadrature of the relation

$$\left.\begin{aligned} \frac{d\ln x}{dF} &= -\frac{D(F,Z)}{Q(F,Z)} \\ \text{or}\qquad \frac{d\ln x}{dZ} &= -\frac{1-F}{Z}\frac{D(F,Z)}{P(F,Z)}, \end{aligned}\right\} \tag{10}$$

where $x \equiv r/r_n$. The velocity and temperature profiles are determined directly from the definitions of F and Z in (1),

$$\frac{u}{u_n} = x\frac{F}{F_n}, \quad \frac{T}{T_n} = \left(\frac{a}{a_n}\right)^2 = x^2\frac{Z}{Z_n}, \tag{11}$$

while the so-called adiabatic integral yields the density profiles

$$\frac{\rho}{\rho_n} = \left[\frac{Z}{Z_n}\left(\frac{1-F}{1-F_n}\right)^{\lambda/(j+1)} x^{\lambda}\right]^{-(j+1)/[(j+1)(\gamma-1)+\lambda]}.$$

The pressure profiles are then determined from the perfect gas equation of state

$$\frac{p}{p_n} = \frac{\rho}{\rho_n}\frac{T}{T_n}.$$

3. Properties of the phase plane

Of all the possible boundary conditions for blast waves propagating into a uniform atmosphere, the point CJ_∞ represents a special case, in that, for a specific value of $\lambda = \lambda_D$, it is a singularity. One can thus have two families of integral curves:

Family I. Those associated with the point CJ_∞ for all values of λ.

Family II. Those associated with the point CJ_∞ for a fixed value of $\lambda = \lambda_D$.

The integral curves of family I are of the same kind as those with the strong shock condition, for which complete solutions were given in our previous paper (Oppenheim *et al.* 1972). Curves belonging to family II are of the same type as those in figure 1 and those for $\lambda = 0$, $\lambda = \frac{2}{5}$ and $\lambda = j+1-w$ are described in the text of Sedov (1959). Their particular significance in the present case is associated with the singular nature of the CJ_∞ point for $\lambda = \lambda_D$. Since the singularity then coincides with the boundary condition, one has a degenerate solution represented by the CJ_∞ point itself.

Curves of family I, covering the full range of λ in the case of $j = 2$ and $\gamma = 1{\cdot}4$, are plotted in the phase plane in figure 2. Continuous lines refer to physically meaningful cases of blast waves bounded by a single discontinuity. Broken lines represent solutions which are, in this respect, meaningless since they cross the sonic ($D = 0$) line and the profiles of gasdynamic parameters to which they correspond thereby become double-valued functions of the space and self-similarity co-ordinate x. As in figure 1, the directions associated with the increase of this co-ordinate are marked by arrows. Since, for all the integral curves, the point CJ_∞ represents conditions at the front, all the curves with arrows pointing towards this point represent explosions, while those having arrows pointing away correspond to implosions.

The point of intersection of an integral curve of family I with the $D = 0$ line is, in effect, a singularity, denoted by the letter A. For $\lambda = 0$, corresponding to a constant-velocity wave, the singularity A is located at $F = 0$, $Z = 1$. As λ increases, A travels along the $D = 0$ line as shown on figure 1. Finally, at

$$\lambda = \lambda_D = \tfrac{2}{3}j\gamma/(\gamma+1) \tag{12}$$

A coincides with the point CJ_∞. For a gas with specific heat ratio $\gamma = 1{\cdot}4$, one thus has specifically

$$\lambda_D = \begin{cases} 0 & \text{for } j = 0, \\ \frac{7}{18} & \text{for } j = 1, \\ \frac{7}{9} & \text{for } j = 2. \end{cases}$$

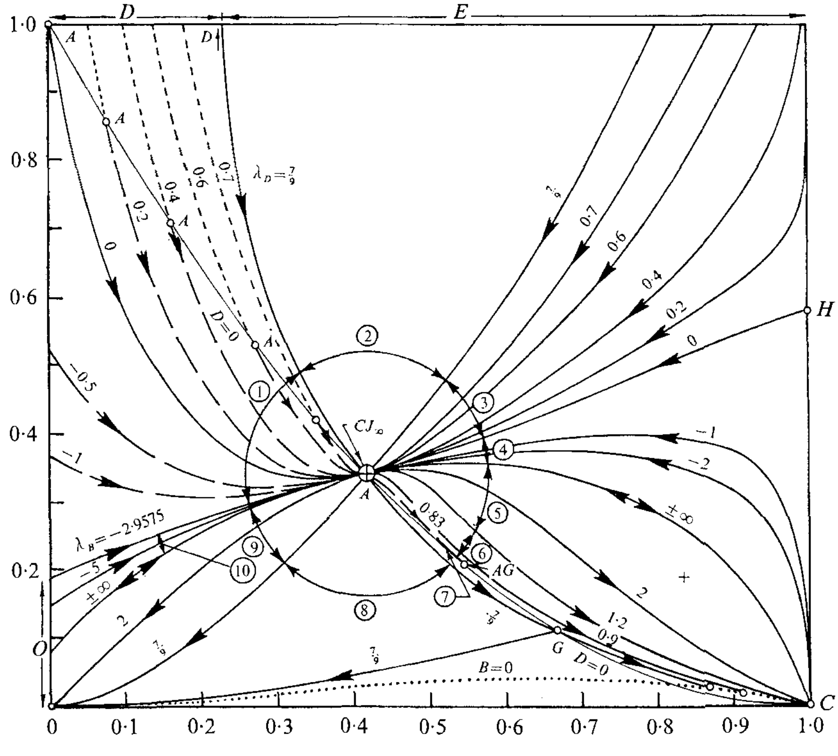

FIGURE 2. Phase plane for family I. Solutions for all values of the decay parameter λ ($j = 2$, $\gamma = 1{\cdot}4$).

The singularity at A is, as a rule, a node. As was shown in our previous paper, its position is given by the intersection of the $Q = 0$ and $P = 0$ lines on the $D = 0$ line. It has a conjugate singularity at the point G, which is always a saddle point. The two are coincident at

$$\lambda = \lambda_{AG} = j\gamma[1-(\tfrac{1}{2}\gamma)^{\frac{1}{2}}/1-\tfrac{1}{2}\gamma]^2, \tag{13}$$

the position of point AG being also shown in figure 2. The $Q = 0$ and $P = 0$ lines can intersect again away from the $D = 0$ line, forming a third singularity B, which, as a rule, is a node. In figure 2 the locus of points B is denoted by a dotted line.

At $Z = \infty$ the integral curves attain either a fixed singularity E, representing conditions at the 'hot' piston face, for which $F_E = 1$, or a singularity D for which

$$F_D = \lambda/(j+1)\gamma. \tag{14}$$

For a gas with the specific heat ratio $\gamma = 1{\cdot}4$, while $\lambda = \lambda_D$, this yields specifically

$$F_D = \begin{cases} 0 & \text{for } j = 0, \\ \frac{5}{36} & \text{for } j = 1, \\ \frac{5}{54} & \text{for } j = 2. \end{cases}$$

Finally, at $Z = 0$ the integral curves can be associated with the singularity O at the origin, i.e. $F = 0$, or with the singularity C at $F = 1$, representing the conditions at the 'cold' piston face.

4. Integral curves of family I

The integral curves in figure 2 are grouped into a number of classes (indicated by the numbers in circles), depending on the particular singularity at which they terminate. Thus the curves in sector 1 are associated with singularity D at $Z = \infty$. Since they also have to pass through singularity A, they cannot be in their entirety physically meaningful, except for the case $\lambda = 0$, corresponding to the well-known solution for a constant-velocity Chapman–Jouguet detonation (Sedov 1959). This class of solutions is bounded on one side by $\lambda = \lambda_B$, for which the integral curve terminates at singularity B, and on the other side by $\lambda = \lambda_D$, the solution corresponding to a decaying 'free' (i.e. unsupported) spherical blast wave bounded by the CJ_∞ detonation front.

For family I the condition $\lambda = \lambda_D$ delineates regions in the phase plane devoid of integral curves, namely sectors 2 and 8. Sector 2 is bounded on the right-hand side by the curve corresponding to a decaying blast wave driven by a 'hot' piston at singularity E whose front has a trajectory in the time–space domain identical to that of the 'free' wave just described. Associated with the same singularity E are all the integral curves in sector 3. This class is in turn bounded by the line corresponding to $\lambda = 0$, the solution for a constant-velocity piston-driven wave. Sector 4 contains integral curves representing 'cold' piston-driven accelerating explosions, and includes all the negative values of λ down to $\lambda = -\infty$. As shown by Oppenheim *et al.* (1972) the curve for $\lambda = -2$ can be considered as the limiting case of an exponential front trajectory, and that for $\lambda = \infty$ as one corresponding to a logarithmic front trajectory.

Sector 5, between $\lambda = +\infty$ and $\lambda = \lambda_C = (j+1)(\gamma-1)$ ($\lambda = 1{\cdot}2$ for $j = 2$, $\gamma = 1{\cdot}4$), embodies 'cold' (zero temperature) piston-driven decelerating implosions, except for the limiting case $\lambda = \lambda_C$, which corresponds to a piston at a finite temperature. Curves in sector 6 terminate at singularity B and represent, therefore, decaying implosions associated with infinite particle velocity and infinite sound speed. Sector 7 near CJ_∞ starts with the case $\lambda = \lambda_{AG}$, which again cannot be in its entirety physically meaningful since the integral curve intersects the $D = 0$ line. All the other curves in this sector intersect the $D = 0$ line between CJ_∞ and AG, and are thus also physically unrealistic.

Sector 8, as has already been pointed out, is devoid of integral curves of family I. Sector 9 consists of curves representing decaying implosions associated with the condition of zero particle velocity and zero sound speed at infinity. Finally, curves in sector 10 correspond to accelerating explosions with zero particle velocity and zero sound speed at the centre. Blast waves in this regime differ from all the other physically meaningful cases discussed so far, in that they contain a region of negative particle velocity, for which $u \to 0$ as $t \to \infty$.

5. Integral curves of family II

Integral curves of family II for $j = 2$ and $\gamma = 1{\cdot}4$ are plotted in the phase plane in figure 3; all of them correspond in this case to $\lambda = \lambda_D = \frac{7}{9}$. They are divided into four sets by the axes of singularity A, which is now located at the point CJ_∞.

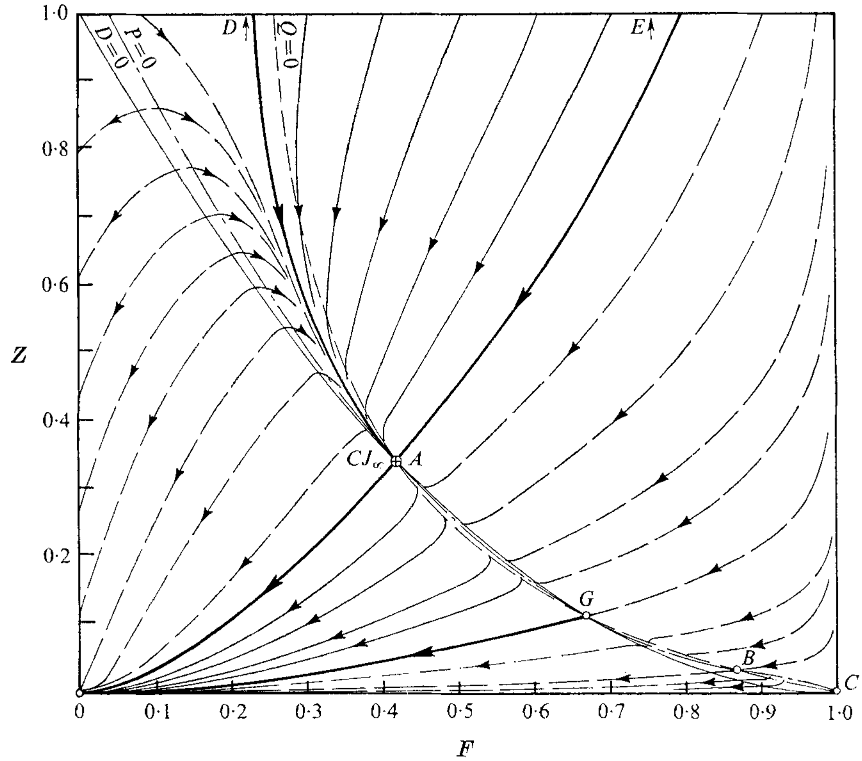

FIGURE 3. Phase plane for family II ($j = 2$, $\lambda = \frac{2}{3}j\gamma/(\gamma+1)$, $\gamma = 1{\cdot}4$).

In figure 3, the branch passing through points G and B has been omitted for the sake of clarity, the physical significance of all the axes having been discussed in the description of figure 2. It is by means of these axes, in fact, that sectors 2 and 8, which are devoid of integral curves in figure 2, are delineated.

In figure 3 the curves in these sectors are distinguished from the rest by the fact that they do not intersect the sonic line $D = 0$. Of all the solutions shown in figure 3 they are therefore the only ones corresponding to physically meaningful cases, thus filling up neatly the gap in figure 2, without any overlap. As has been already pointed out, the co-ordinates of singularity A, situated now at the point CJ_∞, in this case represent by themselves a physically meaningful degenerate solution. The conjugate to singularity A at G is, as was pointed out earlier, a saddle point. In particular its axes represent physically meaningful solutions. One of these is the already described curve AGB, which is at the same time an axis for A, as well as one for B. Since at G it crosses the $D = 0$ line, a branch of the other axis of G, the curve GO, can be also considered as part of a solution. It bounds, in fact, the class of physically meaningful integral curves in sector 8. Thus G admits a double-branched solution, one branch continuing through towards singularity B, while the other has a discontinuity at G and terminates at O.

6. Gasdynamic parameters

Space profiles of the gasdynamic parameters u, ρ, p and $T \propto p/\rho$ corresponding to all the physically meaningful solutions represented on figures 2 and 3 are shown in figures 4, 5, 6 and 7 respectively. The letters labelling the various singularities that appear on these diagrams correspond to the same conditions as in the description of the intergal curves on the phase plane (see also Oppenheim *et al.* 1972). Thus, in figure 4, points labelled E represent conditions at the 'hot piston face', H corresponds to the constant-velocity piston, while C represents conditions at the 'cold piston face'. As is evident from the diagram, when the value of λ decreases the velocity of the piston increases. Figures 4–7 represent primarily the gasdynamic profiles of solutions of family I. They contain just a couple of representative cases of family II, while, on the logarithmic plots of these figures, the degenerate solutions appear as straight lines.

7. Conclusions

To sum up, we have explored all the possible self-similar solutions for blast waves that can be associated with the strong Chapman–Jouguet condition CJ_∞, that is, one corresponding to an infinite pressure ratio at the front. It is our claim that, in view of the proximity of the Chapman–Jouguet points for some representative chemical systems to the point CJ_∞ in the phase plane, such solutions have a distinct possibility of being physically meaningful, although they admit the possibility of the existence of Chapman–Jouguet detonations of variable velocity, which from the classical point of view is paradoxical.

However, at this juncture, we offer the results of our studies just as an interesting case in blast wave theory for two reasons. On one hand, they represent a limit for the whole class of physically meaningful solutions of self-similar blast waves which are bounded by fronts associated with the deposition of energy. On the other, they exhibit some unique properties owing to the fact that a singularity may coincide with the point representing the boundary conditions. This gives rise to two families of solutions, including the degenerate case of one corresponding to this point by itself.

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Appendix. Planar waves

The anomaly associated with the CJ_∞ point acquires a peculiar feature in the case of planar waves for then, according to (12), $\lambda_D = 0$, that is the front velocity is constant. At the same time, however, the $D = 0$ line loses its significance as a locus of singularities. Since, under such circumstances, this case becomes quite different from those of other geometries, it is here considered in particular.

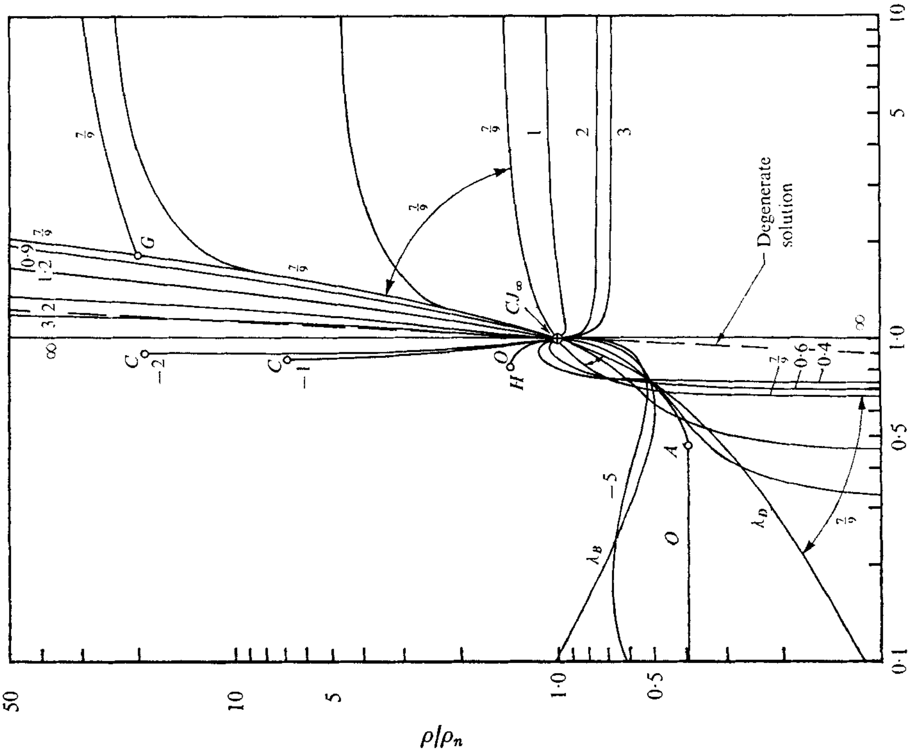

FIGURE 5. Space profiles of densities ($j = 2$, $\gamma = 1 \cdot 4$).

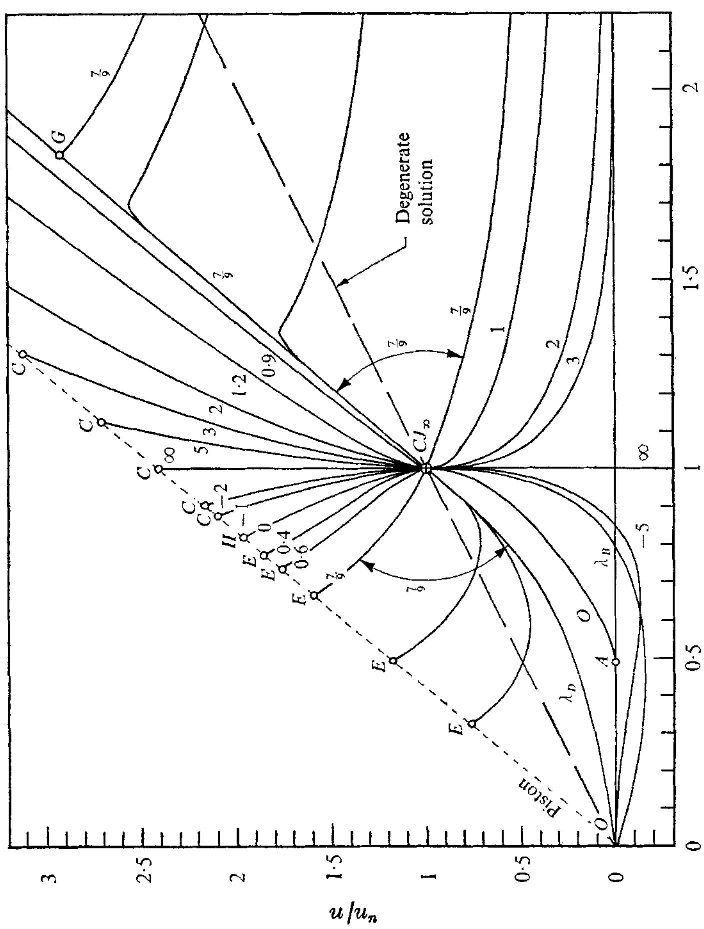

FIGURE 4. Space profiles of particle velocities ($j = 2$, $\gamma = 1 \cdot 4$).

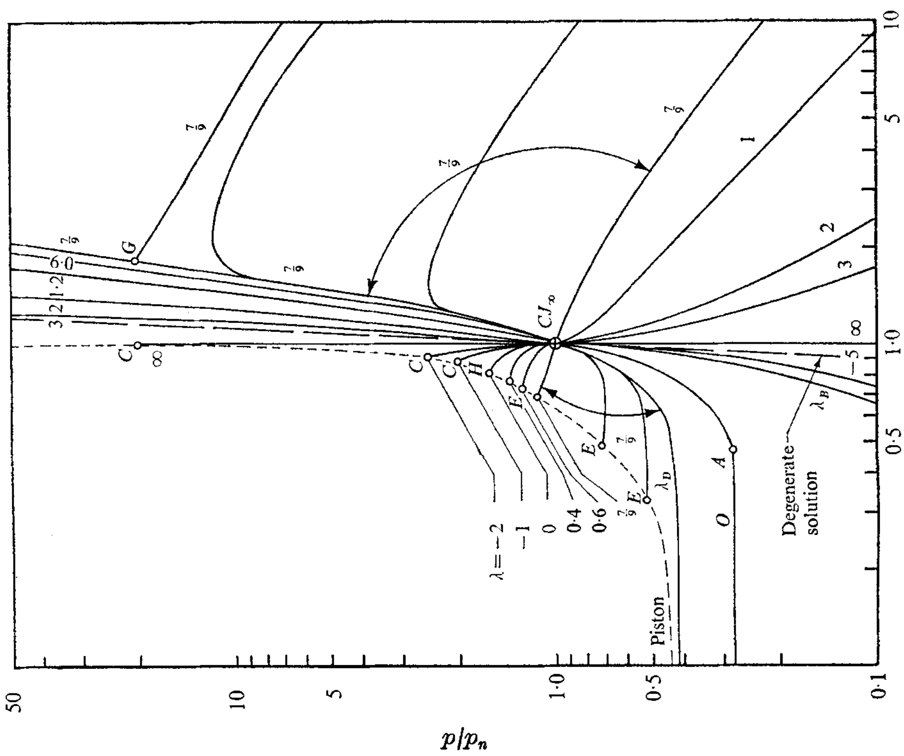

FIGURE 6. Space profiles of pressures ($j = 2$, $\gamma = 1{\cdot}4$).

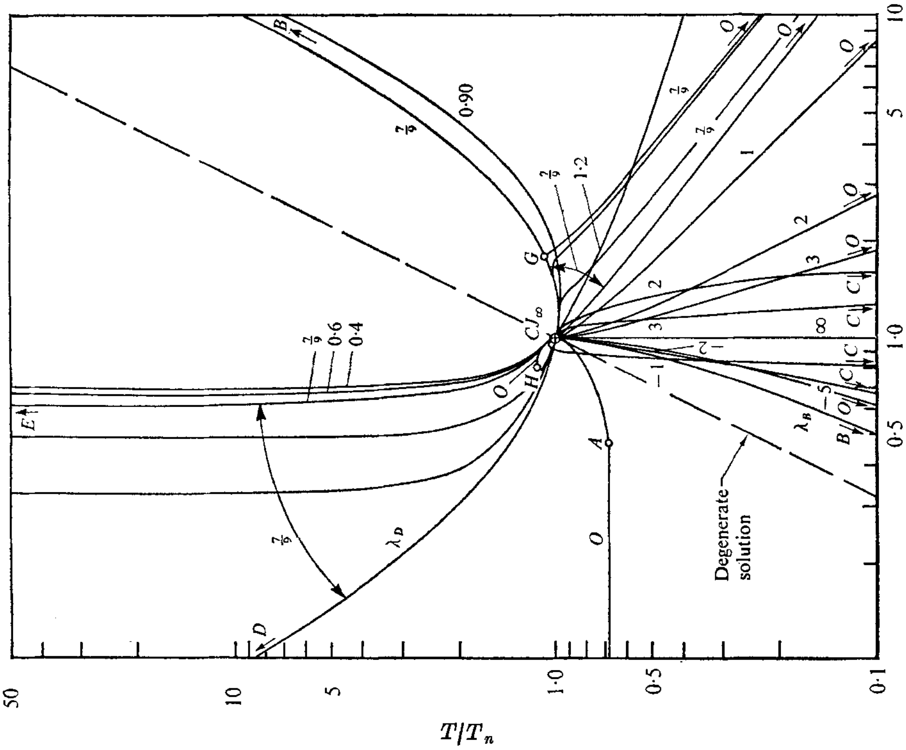

FIGURE 7. Space profiles of temperatures ($j = 2$, $\gamma = 1{\cdot}4$).

In general, for $j = 0$, (3) and (4) become, respectively,

$$Q(F, Z) = FD - (\lambda/\gamma) Z \tag{A 1}$$

and
$$P(F, Z) = 2(1-F) D + \lambda(D - (\gamma - 1/\gamma) Z). \tag{A 2}$$

As before, there are two classes of solutions, one corresponding to piston-driven waves and the other to point explosions.

For the first, $D \neq 0$ and (2), using (A 1) and (A 2) with $\lambda = 0$, reduces to

$$\left.\begin{aligned} dZ/dF &= 2Z/F, \\ Z &= Z_n(F/F_n)^2. \end{aligned}\right\} \tag{A 3}$$

whence

At the same time (10) becomes

$$\left.\begin{aligned} d\ln x/dF &= -1/F, \\ x &= F_n/F. \end{aligned}\right\} \tag{A 4}$$

whence

The information on the flow field is then completed by observing that, by combining the above with (11), one gets

$$\left.\begin{aligned} u &= w_n F_n = \text{constant}, \\ a &= w_n Z_n^{\frac{1}{2}} = \text{constant}. \end{aligned}\right\} \tag{A 5}$$

For the second class of solutions $D = 0$. In this case after substitution of (A 1) and (A 2) into (2) with $D = 0$, the application of l'Hospital's rule yields

$$\left.\begin{aligned} dZ/dF &= -2(1-F), \\ Z &= (1-F)^2. \end{aligned}\right\} \tag{A 6}$$

whence

Equation (A 6) demonstrates that the $D = 0$ condition satisfies the differential equation, and that the $D = 0$ line is therefore an integral curve. It represents, in fact, the well-known family of Riemann solutions associated with Chapman–Jouguet detonation fronts (see e.g. Stanyukovich 1960, chap. VIII). The Riemann invariant can be expressed in reduced co-ordinates as follows:

$$F - \frac{2}{\gamma - 1} Z^{\frac{1}{2}} = \frac{k}{x}. \tag{A 7}$$

By eliminating Z by the use of (A 6) and evaluating the constant k from the boundary condition $F = F_n$ at $x = 1$ one gets

$$x = \frac{1 - \frac{1}{2}(\gamma + 1) F_n}{1 - \frac{1}{2}(\gamma + 1) F} \tag{A 8}$$

while, with the use of the definitions of (1),

$$\left.\begin{aligned} u &= w_n \left[F_n + \frac{2}{\gamma + 1}(x - 1)\right] \\ a &= w_n \left[\frac{1}{\gamma + 1}\{2 + (\gamma - 1)x\} - F_n\right]. \end{aligned}\right\} \tag{A 9}$$

and

The boundary conditions for solutions represented by the $D = 0$ line are related to the front Mach number as follows:

$$F_n = (1 - y)/(\gamma + 1), \quad Z_n = (1 - F_n)^2, \tag{A 10}$$

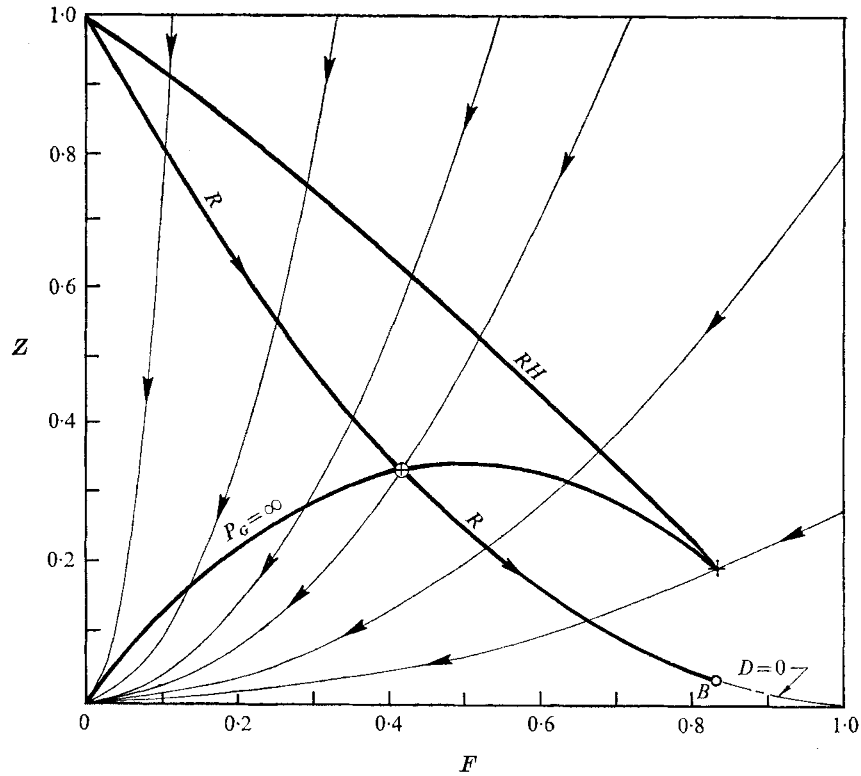

FIGURE 8. Phase plane for planar waves with constant front velocity ($j = 0$, $\lambda = 0$, $\gamma = 1{\cdot}4$).

where $y \equiv 1/M_n^2$. y is, in turn, related to the pressure ratio at constant density across the detonation (P_G), i.e.

$$P_G = 1 + \frac{\gamma}{\gamma+1}\frac{(1-y)^2}{y}. \tag{A 11}$$

Salient properties of the phase plane for the case of $j = 0$ are depicted in figure 8. It is of interest to compare this figure with figure 1. The only difference between the two cases is that now $j = 0$ while before $j = 2$. The main family of the integral curves for $j = 0$ are simple parabolae with their apices at the origin of the phase plane. As before the regime of boundary conditions is bounded by the RH line and the curves representing the conditions of $D = 0$ and $P_G = \infty$. Unlike the previous case, however, the direction of increasing radius does not change across the $D = 0$ line. Thus to the right of the point representing the boundary condition the integral curves represent piston-driven blast waves. To the left, however, they correspond to a new class of implosions which did not exist previously. On the other hand, all the integral curves representing point explosions bounded by a Chapman–Jouguet detonation collapse into the $D = 0$ line itself, the locus of the Chapman–Jouguet conditions. Since again the radius increases monotonically with increasing F, the segment of the $D = 0$ line above the point representing the boundary condition corresponds to an explosion while that below corresponds to an implosion. The latter terminates at B, corresponding to infinite radius. This point is fixed by the intersection of the loci of the $B = 0$ and $D = 0$ conditions (see figure 2), its co-ordinate being specified by $F_B = 2(\gamma+1)$, as is quite obvious from (A 8). In contrast to this the CJ_∞ point becomes devoid of all the interesting properties it possesses in the cases of non-planar geometry.

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